Let f and g be functions with overlapping domains. Then:

1. Sum: $(f+g)(x) = f(x) + g(x)$

2. Difference: $(f-g)(x) = f(x) - g(x)$

3. Product: $(fg)(x) = f(x) \bullet g(x)$

4. Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Examples: Let $f(x) = x^2 + 2x$ and $g(x) = 2x + 1$.

a) $(f+g)(x) = f(x) + g(x) = x^2 + 2x + 2x + 1 = \boxed{x^2 + 4x + 1}$

b) $(f-g)(x) = x^2 + 2x - \boxed{2x + 1} = \boxed{x^2 - 1}$

c) $(fg)(x) = (x^2 + 2x)(2x + 1) = 2x^3 + 5x^2 + 2x$

d) $\left(\frac{f}{g}\right)(x) = \frac{(x^2 + 2x)}{(2x + 1)}$

Compositions of Functions

$$(f \circ g)(x) = f(g(x))$$

Ex: $f(x) = 3x^2 + 2$ **and** $g(x) = 2x$

Find: $f(g(2))$ $g(2) = 2(2) = \boxed{4}$ ← this output is f's input

$f(4) = 3(4)^2 + 2 = \boxed{50}$

$f(g(x))$ $g(x) = \boxed{2x}$ ← input for f

$f(2x) = 3(2x)^2 + 2$

$= 3(4x^2) + 2$

$= \boxed{12x^2 + 2}$

Find $(f \circ g)x, (g \circ f)x$ **and their domains.**

$f(x) = 3x + 5,\ g(x) = \sqrt{x+2}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x+2})$

$f(g(x)) = 3\sqrt{x+2} + 5$

$x + 2 \geq 0$

$x \geq -2$

D: $[-2, \infty)$

$(g \circ f)(x) = g(f(x))$

$= g(3x+5) = \sqrt{3x+5+2}$

$g(f(x)) = \sqrt{3x+7}$

$3x + 7 \geq 0$

$3x \geq -7$

$x \geq \frac{-7}{3}$

D: $\left[\frac{-7}{3}, \infty\right)$

Decomposing a function

Ex. Express $h(x)=\dfrac{1}{\sqrt{3x+1}}$ **as a composition of two functions.**

one ans:

$g(x)=\dfrac{1}{\sqrt{x}}$ $\quad f(x)=3x+1$ $\quad g(f(x))=\dfrac{1}{\sqrt{3x+1}}$

$g(x)=\dfrac{1}{x}$ $\quad g(f(x))=\dfrac{1}{\sqrt{3x+1}}$

or: $f(x)=\sqrt{3x+1}$