1.5 Inverse Functions

Interchanging the x and y values of each ordered pair that belongs to a function will produce its inverse function.

Ex.: $f(x) = 3x - 5$

Inverse: $f^{-1}(x) = \dfrac{x+5}{3}$

x	0	1	2	5
y	-5	-2	1	10

x	-5	-2	1	10
y	0	1	2	5

$f(x) =$ 3 times x, subtract 5

$f^{-1}(x) =$ x plus 5, divided by 3

(read orig. problem backwards, undoing operations as you go.)

Now, find:

$$f\left(f^{-1}(x)\right) = \cancel{3}\left(\frac{x+5}{\cancel{3}}\right) - 5 = x + 5 - 5 = \boxed{x}$$

$$f^{-1}\left(f(x)\right) = \frac{3x - 5 + 5}{3} = \frac{3x}{3} = \boxed{x}$$

If both compositions give the solution x, then the functions are inverses of each other.

To find the inverse of a function algebraically, interchange x and y, then solve for y.

Ex: $f(x) = 3x^3 + 1$

$y = 3x^3 + 1$

$x = 3y^3 + 1$

$x - 1 = 3y^3$

$\frac{x-1}{3} = y^3$

$\boxed{\sqrt[3]{\frac{x-1}{3}} = y = f^{-1}(x)}$

Check:

$f(f^{-1}(x)) = 3\left(\sqrt[\cancel{3}]{\frac{x-1}{3}}\right)^{\cancel{3}} + 1 = \cancel{3}\left(\frac{x-1}{\cancel{3}}\right) + 1 = \boxed{x}$

$f^{-1}(f(x)) = \sqrt[3]{\frac{3x^3+1-1}{3}} = \sqrt[3]{\frac{\cancel{3}x^3}{\cancel{3}}} = \sqrt[3]{x^3} = \boxed{x}$

Not all functions have inverses.

Ex: $f(x) = x^2 - 2$

$$y = x^2 - 2$$

$$\downarrow$$

$$x = y^2 - 2$$

$$x + 2 = y^2$$

$$\boxed{\pm\sqrt{x+2} = y}$$

2 answers, so the inverse is <u>not</u> a function

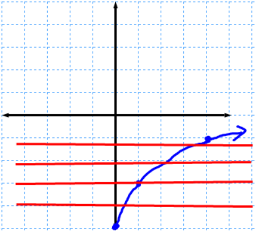
* Functions that have inverses are one-to-one functions.

There are 2 ways to check to see if a function has an inverse. (is one to one?)

1. **Algebraically: If f(a) = f(b), then a = b.** (Don't do)
2. **Horizontal line test with original function.**

(If any horiz. line passes through a graph only once, the function has an inverse.)

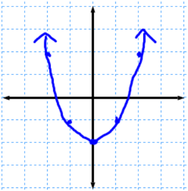
Ex: $f(x) = 2\sqrt{x} - 5$



each line passes through the graph once, so the function has an inverse.

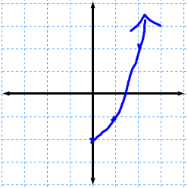
* $f^{-1}(x) = \left(\frac{x+5}{2}\right)^2, \quad [-5, \infty)$

Ex: $f(x) = x^2 - 2$



Can you cut off $\frac{1}{2}$ of the parabola so the function has an inverse?

(either side will do)



Ex: $f(x) = x^2 - 2$, $[0, \infty)$

you must restrict the domain.

* $f(x) = x^2 - 2$, $(-\infty, 0]$ works too!