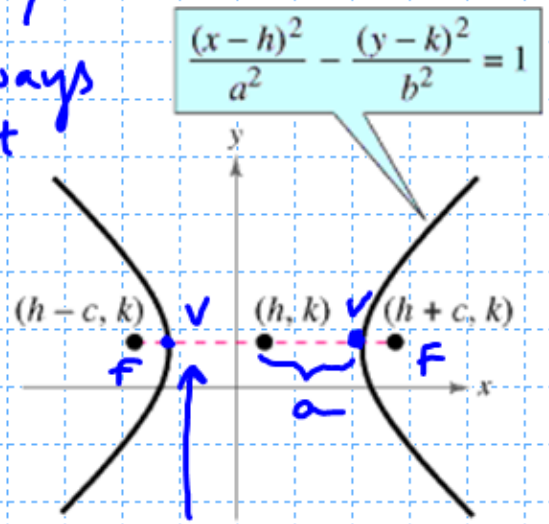
10.3 Hyperbolas

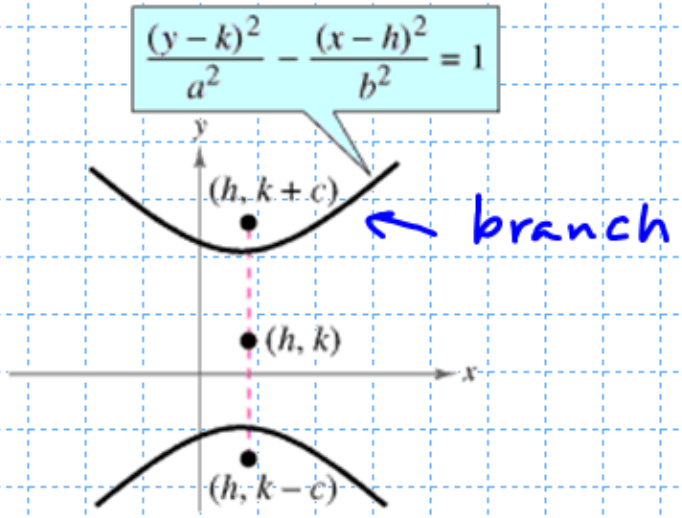
A **hyperbola** is the set of all points (x, y) in a plane the difference of whose **distances** from two distinct fixed points (foci) is constant.

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \qquad \frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

always first



transverse axis (2a)



branch

To find the Foci: $c^2=a^2+b^2$

Eccentricity is $e=\frac{c}{a},\ e>1$

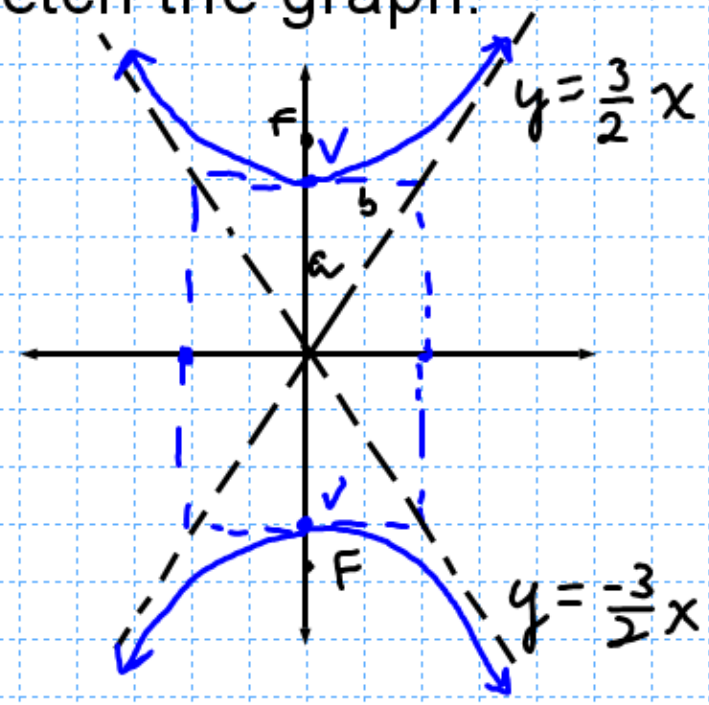
Example 1: Find the center, vertices, asymptotes and foci of the hyperbola given by $4y^2 - 9x^2 = 36$ and sketch the graph.

$\frac{4y^2}{36} - \frac{9x^2}{36} = \frac{36}{36}$

$\frac{y^2}{9} - \frac{x^2}{4} = 1$

$a^2 = 9$, $a = \pm 3$

$b^2 = 4$, $b = \pm 2$



Center: $(0, 0)$

Vertices: $(0, 3), (0, -3)$

Foci: $c^2 = 9 + 4$

$c^2 = 13$

$c = \pm\sqrt{13}$

$(0, \sqrt{13})$

$(0, -\sqrt{13})$

Asymptotes: $y = mx + b$

$y = \frac{3}{2}x + 0$

$y = \frac{-3}{2}x$

Example 2: Find the center, vertices, foci, and asymptotes of $4x^2 - 9y^2 - 24x - 72y - 72 = 0$ and sketch the graph.

$4x^2 - 24x - 9y^2 - 72y = 72$

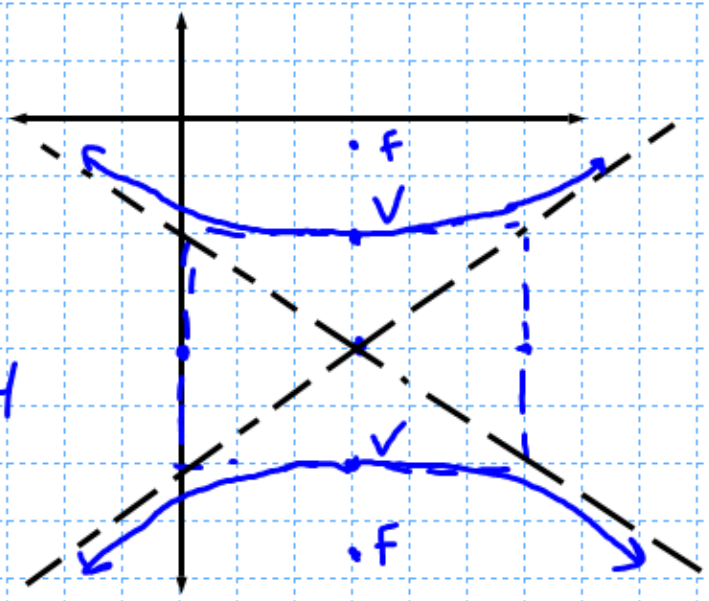
$4(x^2 - 6x + 9) - 9(y^2 + 8y + 16) = 72$ $+36$ -144

$\frac{4(x-3)^2}{-36} - \frac{9(y+4)^2}{-36} = \frac{-36}{-36}$

$-\frac{(x-3)^2}{9} + \frac{(y+4)^2}{4} = 1$

$\frac{(y+4)^2}{4} - \frac{(x-3)^2}{9} = 1$

$\underbrace{4}_{a^2}$



center $(3, -4)$

Vertices: $(3, -2), (3, -6)$

Foci: $c^2 = 9 + 4 = 13$, $c = \pm\sqrt{13}$

$(3, -4 \pm \sqrt{13})$

To find the equations of the asymptotes, use $y = \pm mx + b$. Use the rectangle to find the slope ($m = \pm\frac{b}{a}$ if the transverse axis is horizontal, $m = \pm\frac{a}{b}$ if the transverse axis if vertical). If the y-intercept is difficult to see on the y-axis, replace *x* and *y* with the coordinates of the center and solve for *b*.

$y = \frac{2}{3}x - 6$ $\quad$ $y = -\frac{2}{3}x - 2$

Day 2

Example 1: Find the equation in standard form of the hyperbola with vertices (3, 0), (3, -6) with asymptotes $y = x - 6$, $y = -x$.

$m = 1$

center $(3, -3)$

use

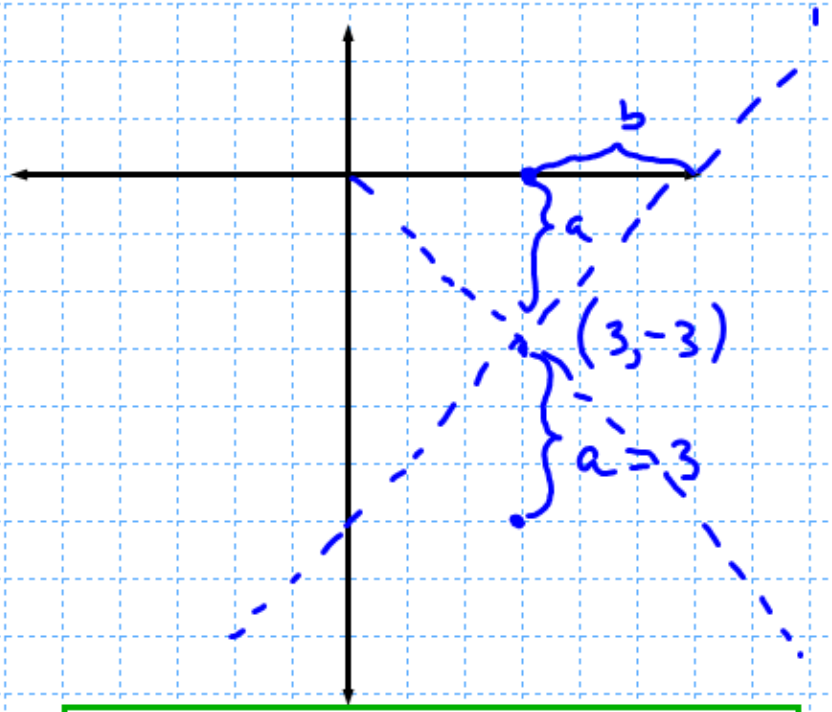
$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$\frac{(y+3)^2}{9} - \frac{(x-3)^2}{b^2} = 1$$

use the slope:

$$m = \frac{a}{b} \qquad 1 = \frac{3}{b}$$

$$b = 3$$



$$\frac{(y+3)^2}{9} - \frac{(x-3)^2}{9} = 1$$

Example 2: Classify each equation as a circle, hyperbola, ellipse or parabola.

$3x^2 - 2y^2 + 4y - 3 = 0$ hyperbola

$2y^2 - 3x + 2 = 0$ parabola

$x^2 + 4y^2 - 2x - 3 = 0$ ellipse

$x^2 - 2x + 4y - 1 = 0$ parabola

$5x^2 + 5y^2 + 25x - 30y - 16 = 0$ circle

equal