12.1 The Concept of a Limit

Example 1: Find $f(4)$ if $f(x) = (3x - 2)$.

Answer: $f(4) = 3(4) - 2 = 10$

Let's look at what happens to $f(x)$ as we get closer and closer to x = 4:

x	3.9	3.99	3.999	4	4.001	4.01	4.1
F(x)	9.7	9.97	9.997	?	10.003	10.03	10.3

$\rightarrow$ 10 $\leftarrow$

$$\lim_{x \to 4} (3x-2) = 10$$

A limit is the <u>height</u> a function <u>intends</u> to reach at a given x value, whether or not it actually reaches it.

In order for a limit to exist, it must reach the intended height from the left side and the right side of some value of x.

<u>Definition:</u> If *f(x)* becomes arbitrarily close to a unique number *L* as *x* approaches *c* from either side, the limit of *f(x)* as *x* approaches *c* is *L*. This is written as

$$\lim_{x \to c} f(x) = L$$

Limits can be evaluated by using tables, by inspecting graphs or by direct substitution.

Example 2: Use a table to evaluate $\lim_{x\to 2}\frac{(x^2-4)}{(x-2)}$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$F(x)$	3.9	3.99	3.999	?	4.001	4.01	4.1

use table because direct substitution produces $\frac{0}{0}$.

$$\lim_{x\to 2}\left(\frac{x^2-4}{x-2}\right) = 4$$

Example 3: Use a graph to estimate

$$\lim_{x\to 2}\frac{x-2}{\sqrt{x+2}-2} = 4$$

graph, press trace, type in 1.999 & 2.001

Example 4: Evaluate the following limits by direct substitution:

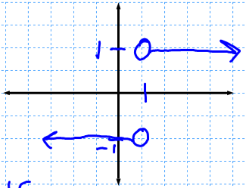
a) $\lim_{x \to 4} \sqrt{x} = \sqrt{4} = 2$

b) $\lim_{x \to 1} x^2 - 2x + 5 = 1^2 - 2(1) + 5 = 4$

c) $\lim_{x \to 0} \dfrac{x+1}{x^2 + 5x - 4} = \dfrac{0+1}{0+0-4} = -\dfrac{1}{4}$

Limits That Fail to Exist

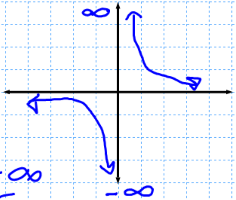
Example 1: $\lim\limits_{x \to 1} \dfrac{|x-1|}{x-1}$



left: limit $= -1$ $\quad -1 \neq 1$ $\quad$ DNE

right: limit $= 1$

Example 2: $\lim\limits_{x \to 0} \dfrac{1}{x}$



left: limit $= -\infty$

right: limit $= \infty$ $\quad -\infty \neq \infty$

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