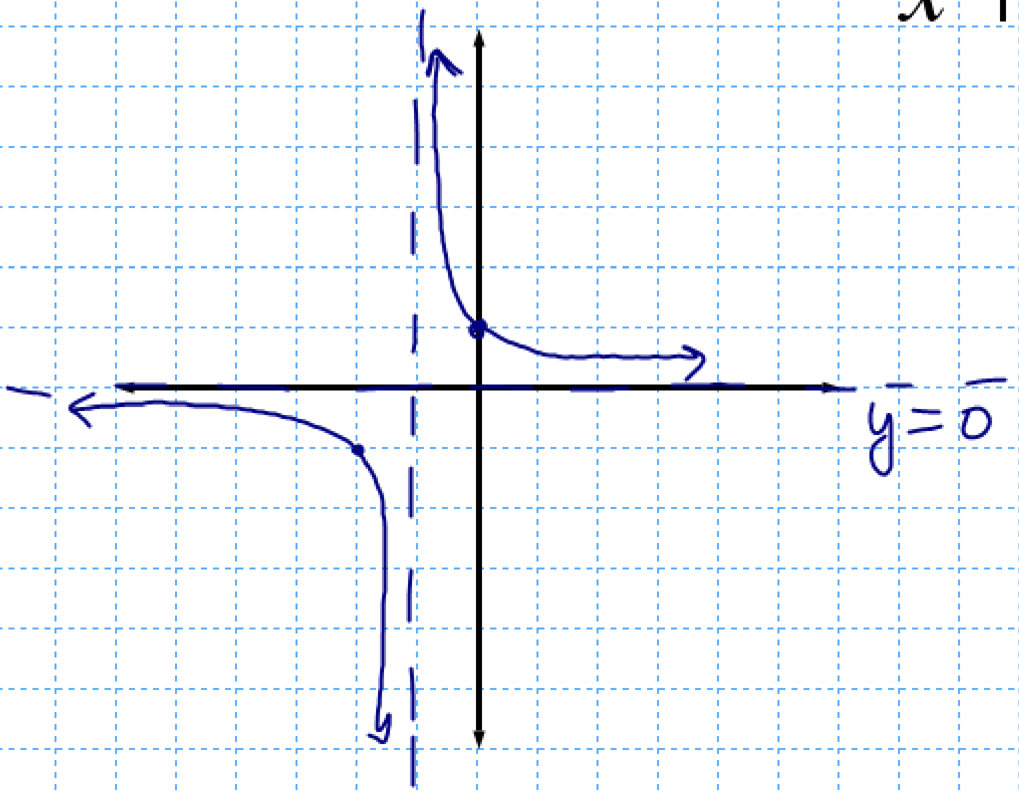
12.4 Limits at Infinity and Limits of Sequences

Find the limit by graphing: $\lim_{x \to \infty} \frac{1}{x+1} = 0$



(the horizontal asymptote is the limit at infinity)

For the rational function $f(x) = \dfrac{N_x}{D_x} = \dfrac{ax^{n} + \cdots}{bx^{m} + \cdots}$,

$$\lim_{x\to\infty} f(x) = \begin{cases} 0 & \text{if } n < m \\ \dfrac{a}{b} & \text{if } n = m \end{cases}$$

just like section 2.6

If $n > m$, the limit does not exist.

- compare the exponents
- find the asymptote, then you have found the limit at infinity!

Examples:

$$\lim_{x\to\infty} = \frac{x^{1}-1}{x^{2}} = 0$$

$1<2$

$$\lim_{x\to\infty} = \frac{x^{2}-1}{2x^{2}} = \frac{1}{2}$$

$2=2$

$$\lim_{x\to\infty} = \frac{x^{3}}{x^{1}}$$

No Limit

$3>1$

Example: Find the limit of the following sequences:

(use the same rules as limits of infinity)

$$\lim_{n\to\infty} = \frac{3n^{1}}{n^{2}+1} = 0$$

$1 < 2$

$$\lim_{n\to\infty} = \frac{3n^{2}}{n^{2}+1} = 3$$

$2 = 2$

$$\lim_{n\to\infty} = \left[2 + \frac{n\left(n^2-1\right)}{n^4}\right] = \lim_{n\to\infty} 2 + \lim_{n\to\infty} \frac{n^3-n}{n^4}$$

$$= 2 + 0$$

$$= \boxed{2}$$