2.1 Polynomial and Rational Functions

Turn to page 135.

Polynomials are named by their highest degree:

1. $y = c$	Constant
2. $y = 2x + 3$	Linear
3. $y = 5x^2 + 6x + 1$	Quadratic
4. $y = 2x^3 + 5$	Cubic
5. $y = 5x^4 + 2x^3 + x + 1$	4th degree

Let's look at quadratic functions ($y = ax^2 + bx + c$).
Turn to pg. 137-138.

Standard form of a quadratic equation:

$$y = a(x-h)^2 + k,\ vertex: (h,k),\ a \neq 0$$

To convert quadratic functions into standard form, we complete the square.

$y = a(x-h)^2 + k$

Ex.: Write $y = x^2 + 2x - 3$ in standard form.

$y + 3 = x^2 + 2x$

$y + 3 \underline{+1} = x^2 + 2x \underline{+1}$

$y + 4 = (x+1)(x+1)$

$y + 4 = (x+1)^2$

$\boxed{y = (x+1)^2 - 4}$

$\frac{2}{2} = 1$

$1^2 = 1$

vertex: $(-1, -4)$

1. Move the constant to the left side
2. Divide the coef. of x by 2, square it, add the result to both side
3. Factor the right side, simplify the left side
4. Move the constant to the right side.

Ex.: Write $y = x^2 + 3x + \frac{1}{4}$ in standard form.

$$y - \frac{1}{4} = x^2 + 3x$$

$$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$y - \frac{1}{4} + \underline{\frac{9}{4}} = x^2 + 3x + \underline{\frac{9}{4}}$$

$$y + 2 = \left(x + \frac{3}{2}\right)\left(x + \frac{3}{2}\right)$$

factors are $\frac{b}{2}$!!

$$y + 2 = \left(x + \frac{3}{2}\right)^2$$

$$\boxed{y = \left(x + \frac{3}{2}\right)^2 - 2 \qquad \text{vertex: } \left(-\frac{3}{2}, -2\right)}$$

Ex.: Write $y = -4x^2 + 24x - 41$ in standard form.

$y + 41 = -4x^2 + 24x$ ← factor out the coef. of x^2

$y + 41 = -4(x^2 - 6x \quad)$

$\frac{-6}{2} = -3$

$(-3)^2 = 9$

-36

because we added -36 on right side if you distribute -4

$y + 41 + (-36) = -4(x^2 - 6x + 9)$

$y + 5 = -4(x-3)(x-3)$

$y + 5 = -4(x-3)^2$

$$\boxed{\begin{array}{c} y = -4(x-3)^2 - 5 \\ \text{vertex: } (3, -5) \end{array}}$$

<u>Ex.:</u> Find an equation of a parabola in standard form, given that the vertex is (4, -1) and it passes through (2, 3).

h , k   x , y

standard form: $y = a(x-h)^2 + k$

we need a, h & k: $y = a(x-4)^2 - 1$

Use (2, 3) to find a: $3 = a(2-4)^2 - 1$

$3 = a(-2)^2 - 1$

$3 = 4a - 1$

$4 = 4a$

$1 = a$

$\boxed{y = 1(x-4)^2 - 1}$