2.2(b) Finding zeros (x-intercepts):

Ex.: $f(x) = 6x^4 - 33x^3 - 18x^2$

$$0 = 3x^2(2x^2 - 11x - 6)$$

$$0 = 3x^2(2x+1)(x-6)$$

$3x^2 = 0$ | $2x+1=0$ | $x-6=0$
$x^2 = 0$ | $2x = -1$ | $x = 6$
$x = 0$ | $x = \frac{-1}{2}$ |

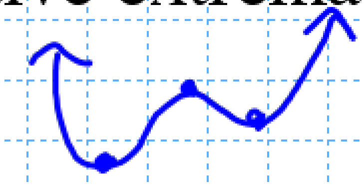
zeros: 0, $-\frac{1}{2}$, 6

$\uparrow$
double

Facts about zeros of polynomial functions:

1. The graph of ***f*** has a most n real zeros.
2. The function of ***f*** has at most n-1 relative extrema (min/max)

   $y = 6x^7 + \cdots$

3. x = a is a zero of the function
4. x = a is a solution of the function f(x) = 0
5. (x-a) is a factor of the polynomial, f(x)
6. (a, 0) is an x-intercept of the graph of ***f***.

Ex.: Sketch the graph of: $f(x) = 6x^4 - 33x^3 - 18x^2$

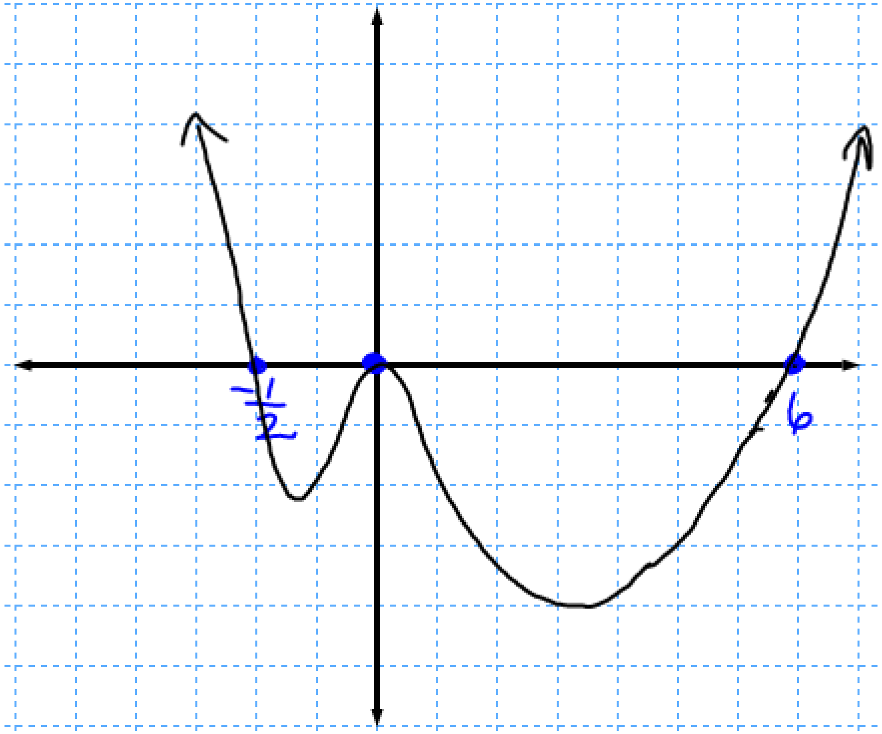
Zeros: $0, -\frac{1}{2}, 6$

↑ double zero

① Plot the zeros

② End behavior ↑∿↗

③ Sketch: pass through single roots, bounce off double roots.



Ex.: Sketch the graph of: $f(x) = x^4 - 4x^2$

① find the zeros

$$0 = x^4 - 4x^2$$

$$0 = x^2(x^2 - 4)$$

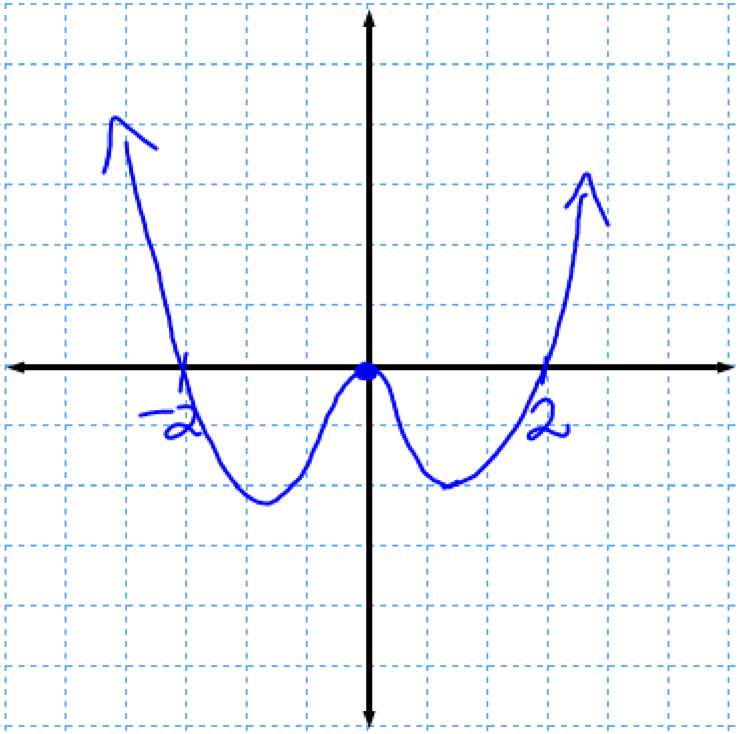
$$0 = x^2(x-2)(x+2)$$

$x^2 = 0$, $x = 0$ | $x = 2$ | $x = -2$

zeros: 0, $\underbrace{2, -2}_{\text{singles}}$

double (bounce)

singles (pass through)



looks like: 

Ex.: Find a polynomial function of degree 3 that has zeros of $0, 2, -\frac{1}{3}$.

① Set each zero $= x$. $\quad x = 0. \quad x = 2 \quad x = -\frac{1}{3}$

② Set each equation $= 0$. $\quad x - 0 = 0 \quad x - 2 = 0 \quad 3x = -1$

$3x + 1 = 0$

③ Write in factored form

$0 = (x-0)(x-2)(3x+1)$

$0 = x(x-2)(3x+1)$

④ Distribute and add like terms.

$0 = x(3x^2 - 5x - 2)$

$$\boxed{f(x) = 3x^3 - 5x^2 - 2x}$$