2.3(a) Polynomial Division

Recall dividing $\dfrac{658}{3}$ using long division:

$$
\begin{array}{r}
219 \\
3\overline{)658} \\
-6 \\
\hline
58 \\
-3 \\
\hline
28 \\
27 \\
\hline
1
\end{array}
$$

219 R. 1

Divide: $\dfrac{2x^3+6x^2-14x+9}{x-1}$

$$
\begin{array}{r l}
 & 2x^2+8x-6 \\
x-1 \,\big) & 2x^3+6x^2-14x+9 \\
 & -(2x^3-2x^2) \downarrow \\
 & \quad 8x^2-14x \\
 & \quad -(8x^2-8x) \\
 & \qquad -6x+9 \\
 & \qquad -(-6x+6) \\
 & \qquad\qquad 3
\end{array}
$$

$\dfrac{2x^3}{x}$

$\dfrac{8x^2}{x}$

$\dfrac{-6x}{x}$

Ans: $\underbrace{2x^2+8x-6}_{\text{quotient}} + \dfrac{3}{\underbrace{x-1}_{\text{divisor}}}$ ← remainder

Divide: $\dfrac{4x^5 - x^3 + 2x^2 - x}{2x+1}$

$$2x^4 - x^3 + x - 1$$

$$2x+1 \,\overline{)\, 4x^5 + 0x^4 - x^3 + 2x^2 - x + 0}$$

$$-(4x^5 + 2x^4)$$

$$-2x^4 - x^3$$

$$-(-2x^4 - x^3)$$

$$0 \quad +2x^2 - x$$

$$-(2x^2 + x)$$

$$-2x + 0$$

$$-(-2x - 1)$$

$$1$$

$\dfrac{4x^5}{2x}$

$\dfrac{-2x^4}{2x}$

$\dfrac{2x^2}{2x}$

$\dfrac{-2x}{2x}$

Ans: $2x^4 - x^3 + x - 1 + \dfrac{1}{2x+1}$

Dividing Polynomials Using Synthetic Division

Divide: $$\frac{2x^3+6x^2-14x+9}{x-1}$$

Shelf #

Set $x-1=0$

$x=1$

$$\begin{array}{r|rrrr} 1 & 2 & 6 & -14 & 9 \\ \text{(add)} & \downarrow & 2(1)\ 2 & 8(1)\ 8 & -6(1)\ -6 \\ \hline & 2 & 8 & -6 & \boxed{3} \end{array}$$

← coef. of the variables

← remainder

$$\boxed{2x^2+8x-6+\frac{3}{x-1}}$$

← put the variables back in, start with one degree less than orig. problem

Divide: $\dfrac{4x^5 \overset{0x^4}{} - x^3 + 2x^2 - x + 0}{2x+1}$

Shelf #

$2x+1=0$

$x=-\frac{1}{2}$

$$\begin{array}{r|rrrrrr} -\frac{1}{2} & 4 & 0 & -1 & 2 & -1 & 0 \\ & \downarrow & -2 & 1 & 0 & -1 & 1 \\ \hline & 4 & -2 & 0 & 2 & -2 & 1 \end{array}$$

$$\boxed{4x^4 - 2x^3 + 2x - 2 + \frac{1}{2x+1}}$$

We can use synthetic division to evaluate functions:

The remainder theorem: If f(x) is divided by x-k, the r=f(k)

Ex: Use the R. theorem to evaluate $f(x) = 4x^2 - 10x - 21$ when x=5

$f(5) = 4(5)^2 - 10(5) - 21 = \boxed{29}$

$$\begin{array}{r|rrr} 5 & 4 & -10 & -21 \\ & \downarrow & 20 & 50 \\ \hline & 4 & 10 & \boxed{29} \end{array}$$

equal!

remainder = f(5) !!

<u>We can also use synthetic division to find zeros of a function.</u>

X = k is a zero if and only if f(k)=0 (i.e.: remainder=0)

$0x^2$

<u>Ex:</u> Is $x = -4$ a zero of $f(x) = x^3 - 28x - 48$?

$$
\begin{array}{r|rrrr}
-4 & 1 & 0 & -28 & -48 \\
 & \downarrow & -4 & 16 & 48 \\
\hline
 & 1 & -4 & -12 & 0
\end{array}
$$

yes! $x = -4$ is a zero.

Find the remaining zeros:

(factor or use quad. formula)

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x = 6, -2$

$\boxed{\text{zeros: } -4, 6, -2}$