2.4 Complex Numbers

Solve: $x^2 + 1 = 0$

$x^2 = -1$ Not possible in the real number system!

The imaginary number system was created in order to solve these types of problems!

Imaginary Numbers: $i = \sqrt{-1}$

$i^2 = -1$

Now we can solve the problem:

$x^2 = -1$

$x = \pm\sqrt{-1}$

$x = \pm i$

Examples:

$\sqrt{-36} = \sqrt{36} \cdot \sqrt{-1} = 6i$

$\sqrt{-7} = \sqrt{7} \cdot \sqrt{-1} = i\sqrt{7}$ or $\sqrt{7}\,i$

$\sqrt{-12} = \sqrt{4 \cdot 3 \cdot (-1)} = 2i\sqrt{3}$

$\sqrt{-7} \cdot \sqrt{-7} = \sqrt{-1} \cdot \sqrt{7} \cdot \sqrt{-1} \cdot \sqrt{7}$

$= i\sqrt{7} \cdot i\sqrt{7}$

$= i^2\sqrt{49}$

$= \boxed{-7}$

$\sqrt{-3} \cdot \sqrt{-5} = i\sqrt{3} \cdot i\sqrt{5} = i^2\sqrt{15} = \boxed{-\sqrt{15}}$

Complex Numbers

$$a + bi$$

↑ real ↑ imaginary

Operations with Complex Numbers $(+, -, \times, \div)$

Add: $(-1+2i)+(3+3i) = 2+5i$

Subtract: $(2-3i)+(-3+7i) = -1+4i$

Multiply: $7i(2-3i)$

$= 14i - 21i^2$

$= 14i - 21(-1)$

$= 14i + 21$

$= 21 + 14i$

$i^2 = -1$

Multiply: $(2+3i)(-6-2i)$

$= -12 - 4i - 18i - 6i^2$

$= -12 - 22i - 6(-1)$

$= -12 - 22i + 6$

$= -6 - 22i$

Multiply: $(1+2i)(1-2i)$

$= 1 - 2i + 2i - 4i^2$

$= 1 + 0 + 4$

$= 5$

Complex Conjugates are two complex numbers in the form $a+bi$ and $a-bi$. Their product is always a real number.

Example: Write the quotient in standard form.

$$\frac{(2-7i)}{(1+i)} \cdot \frac{(1-i)}{(1-i)} = \frac{2-2i-7i+7i^2}{1+i-i-i^2} = \frac{2-9i-7}{1+1} = \boxed{\frac{-5-9i}{2}}$$

Multiply the numerator and the denominator by the conjugate of the denominator.

Patterns of Imaginary Numbers

$i^0 = 1$

$i^1 = i$

$i^2 = -1$

$i^3 = i^2 \cdot i = -i$

$i^4 = i^2 \cdot i^2 = 1$

$i^5 = i^4 \cdot i = i$

$i^6 = i^4 \cdot i^2 = -1$

$i^7 = i^4 \cdot i^3 = -i$

$i^8 = i^4 \cdot i^4 = 1$

$i^9 = i^8 \cdot i = i$

← The pattern repeats at every multiple of 4 on the exponent.

ex: $i^{1093} = i$ because $i^{1092} = 1$

and 1092 is a multiple of 4.