2.5 Finding All Zeros (Real and Complex)

Example: Find all the zeros of

$$f(x) = \overset{q}{x^4} - 3x^3 + \overset{p}{x} - 3$$

$\frac{p}{q} = \pm\left(\frac{1,3}{1}\right)$

3 | 1 −3 0 1 −3
3 0 0 3
1 0 0 1 0 ✓

−1 | 1 0 0 1 0
−1 1 −1
1 −1 1 0 ✓

$x^2 - 1x + 1 = 0$ → use q. formula:

$$x = \frac{1 \pm \sqrt{1 - 4(1)(1)}}{2}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

Zeros: $3, -1, \frac{1 \pm i\sqrt{3}}{2}$

Write as a product of linear factors:

$f(x) = x^4 - 16 = (x^2-4)(x^2+4)$ ← now we can factor using imaginary numbers!

$= (x-2)(x+2)(x-2i)(x+2i)$

$f(x) = x^2 + 9 = (x-3i)(x+3i)$

$f(x) = x^2 + 25 = (x-5i)(x+5i)$

$f(x) = x^2 + 7 = (x-i\sqrt{7})(x+i\sqrt{7})$

$f(x) = x^4 + 13x^2 + 36 = (x^2+9)(x^2+4)$

$= (x+3i)(x-3i)(x+2i)(x-2i)$

Find a third degree polynomial with integer coefficients that has $2, 3+i, 3-i$ as zeros.

1. Set each zero = x: $x=2$, $x=3+i$, $x=3-i$

2. Move the real #s to the left: $x-2=0$ $x-3=i$ $x-3=-i$

3. Square one of the imaginary equations: $(x-3)^2=i^2$ (not needed)

$x^2-6x+9=-1$

$x^2-6x+10=0$

4. Write $x-2$ and $x^2-6x+10$ as factors, then multiply: $(x-2)(x^2-6x+10)$

$=x^3-6x^2+10x-2x^2+12x-20$

$=\boxed{x^3-8x^2+22x-20}$

Use the zero $x = 2i$ to find zeros of (we need 4 total)

$$f(x) = x^4 - x^3 - 2x^2 - 4x - 24$$

- Since complex zeros come in conjugate pairs, $x = -2i$ is also a zero.
- You can get the remaining zeros using synthetic division.

$\frac{p}{q} = \pm\left(\frac{1,2,3,4,6,8}{1}\right)$

3 | 1 -1 -2 -4 -24

$\downarrow$ 3 6 12 -24

1 2 4 8 0 ✓

-2 | 1 2 4 8

-2 0 -8

1 0 4 0 ✓

Zeros: $x = 3, -2, \pm 2i$

$x^2 + 4 = 0$

$x^2 = -4$ (Verifies above info.)

$x = \pm 2i$