

2.7 Graphing Rational Functions

1. Find the y-intercept by evaluating $f(0)$.
2. Find the x-intercept by setting the numerator = 0 and solving for x.
3. Find the vertical asymptotes by setting the denominator = 0 and solving for x.
4. Find the horizontal asymptote by using the following rules: Let

$$f(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n \dots}{b_m x^m \dots}$$

If $n < m$, $y = 0$ is the H.A.

If $n = m$, $y = \frac{a_n}{b_m}$ is the H.A.

If $n > m$, there is no H.A.

5. Plot at least one point between and one point beyond each x-int. and V.A.
6. Connect with smooth curves.

Example: Sketch the graph of the following function:

$$g(x) = \frac{(x-2)}{(x^2 - 2x - 8)}$$

1. x-intercept: $(2, 0)$ $0 = x - 2, x = 2$

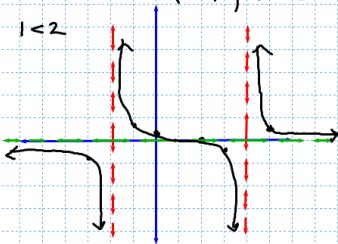
2. y-intercept: $(0, \frac{1}{4})$ $g(0) = \frac{0-2}{0-0-8} = \frac{1}{4}$

3. V.A. $x = -2$
 $x = 4$ $x^2 - 2x - 8 = 0$ $(x-4)(x+2) = 0$
 $x = 4, x = -2$

4. H.A.: $y = 0$ $\frac{x^1}{x^2} \quad 1 < 2$

5. Plot points:

x	y
-3	-.71
-1	.6
3	-.2
5	.42



Example: Sketch the graph of the following function:

$$h(x) = \frac{x}{x^2 + 1}$$

1. x-intercept: $(0, 0)$ $0 = x$

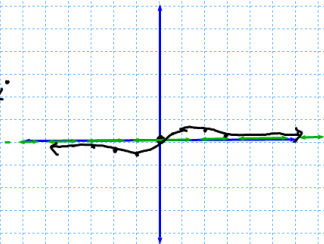
2. y-intercept: $(0, 0)$ $h(0) = \frac{0}{0+1} = 0$

3. V.A.: None $x^2 + 1 = 0$

4. H.A.: $y = 0$ $x^2 = -1$
 $x = \pm i$

5. Plot points:

$$\frac{x^1}{x^2} \quad | < 2$$



x	y
-2	-.4
-1	-.5
1	.5
3	.3

x	y
2	.4
-3	-.3
.5	.4

Special Case of Rational Functions: If the degree of the numerator is exactly one more than the degree of the denominator, the graph of the function has a slant (or oblique) asymptote. Long division is used to find the equation of the slant asymptote. It is of the form $y = q(x)$, where $q(x)$ is the quotient.

Example: Sketch the graph of the following function:

$$0 \quad f(x) = \frac{x^2}{x-2}$$

1. x-intercept: $(0, 0)$ $0 = x^2, x = 0$

2. y-intercept: $(0, 0)$ $f(0) = \frac{0}{0-2} = 0$

3. V.A.: $x = 2$ $x - 2 = 0$
 $x = 2$

4. H.A.: None

$$\frac{x^2}{x^2}$$

$$2 > 1$$

5. S.A.: $y = x + 2$

$$\begin{array}{r}
 x+2 \\
 x-2 \overline{) x^2 + 0x + 0} \\
 \underline{-(x^2 - 2x)} \\
 2x \\
 \underline{-(2x - 4)} \\
 4
 \end{array}$$

Set quotient = y
 $\rightarrow y = x + 2$

6. Plot points:

x	y
-2	-1
-1	3
1	-1
3	9
4	8
5	8.3
6	9

