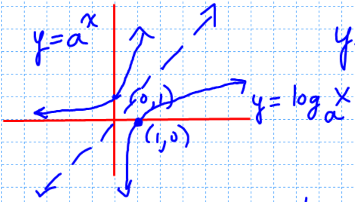
3.2 Logarithmic Functions

Since $y = a^x$ is a one-to-one function, it has an inverse.



$y = a^x$ $\qquad$ $\underbrace{x = a^y}_{\text{inverse}}$

exponent

$$y = \log_a x \quad \text{if and only if} \quad x = a^y$$

base

$$y = \log_a x \quad \text{if and only if} \quad x = a^y$$

Example: Rewrite the following as a logarithm or as an exponent.

1. $2^3 = 8$ $\quad 3 = \log_2 8 \quad$ or $\quad \log_2 8 = 3$

2. $9^{\frac{3}{2}} = 27$ $\quad \frac{3}{2} = \log_9 27$

3. $\log_3 81 = 4$ $\quad 3^4 = 81$

4. $\log_{16} 8 = \frac{3}{4}$ $\quad 16^{3/4} = 8$

Evaluate: ① Set log $= x$ ② Rewrite as an exponent

1. $\log_2 8 = x$

$2^x = 8$ $\qquad 2^x = 2^3$ $\qquad \boxed{x = 3}$

2 2 2

2. $\log_2 0.25 = x$

$2^x = .25 \qquad 2^x = \frac{1}{4} \qquad 2^x = \frac{1}{2^2} \qquad 2^x = 2^{-2}$

$\boxed{x = -2}$

3. $\log_3 81 = x$

$3^x = 81 \qquad 3^x = 3^4 \qquad \boxed{x = 4}$

9 9

3 3 3 3

Properties of Logarithms

1. $\log_a 1 = 0$ because $a^0 = 1$

2. $\log_a a = 1$ because $a^1 = a$

3. $\log_a a^x = x$, and $a^{\log_a x} = x$ (inverse pty)

4. If $\log_a x = \log_a y$, then $x = y$ (one-to-one pty)

Example: Solve for x:

1. $\log_5 x = \log_5 8$ $x = 8$

2. $\log_3 3^{-5} = x$ $x = -5$

Graph $y = \log_2 x$ $\quad 2^y = x$

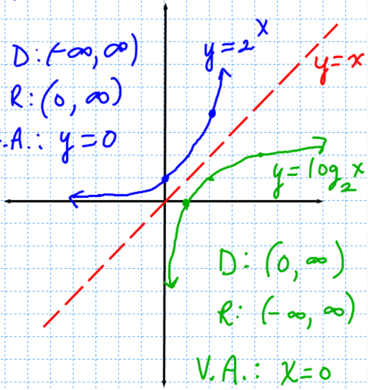
graph the inverse then interchange x and y-values

$y = 2^x$

x	y
0	1
1	2
2	4
-1	$\frac{1}{2}$

$y = \log_2 x$

x	y
1	0
2	1
4	2



D: $(-\infty, \infty)$
R: $(0, \infty)$
H.A.: $y = 0$

D: $(0, \infty)$
R: $(-\infty, \infty)$
V.A.: $x = 0$

The Natural Logarithm – A logarithm where the base is e.

$y = \log_e x$ can be written as $y = \ln x$

All properties of logarithms apply to natural logarithms.

Ex.: Find $\ln e^5$ $= \log_e e^5 = 5$

Find $\ln 1$ $= \log_e 1 = 0$