3.3 Properties of Logarithms

Change of Base Formula

Let a, b, and x be positive real numbers such that $a \neq 1$ and $b \neq 1$. Then

$$\log_a x = \frac{\log_b x}{\log_b a} = \frac{\log x}{\log a} = \frac{\ln x}{\ln a}$$

Example: Evaluate the following:

a) $\log_5 18 = \frac{\log 18}{\log 5} \approx 1.8 \qquad \frac{\ln 18}{\ln 5} \approx 1.8$

b) $\log_2 42 = \frac{\log 42}{\log 2} \approx 5.39 \qquad \frac{\ln 42}{\ln 2} \approx 5.39$

Properties of Logarithms

Since logarithms are exponents, the following properties are similar to exponent properties.

1. $\log_a(uv) = \log_a u + \log_a v$

2. $\log_a \frac{u}{v} = \log_a u - \log_a v$

3. $\log_a u^{n} = n\log_a u$

Example: Expand the logarithmic expression.

a) $\log(2x^3y^4) = \log 2 + 3\log x + 4\log y$

b) $\ln\dfrac{\sqrt{x+5}}{y^2} = \ln \dfrac{(x+5)^{\frac{1}{2}}}{y^2} = \frac{1}{2}\ln(x+5) - 2\ln y$

Example: Condense the logarithmic expression to a single logarithm.

a) $2\log x - 3\log y + \frac{1}{2}\log z$

$$\log \frac{x^2 z^{\frac{1}{2}}}{y^3} = \log \frac{x^2\sqrt{z}}{y^3}$$

b) $\frac{1}{3}(2\ln x - 4\ln y - \ln(z+2))$

$$\frac{1}{3}\left(\ln \frac{x^2}{y^4(z+2)}\right) = \ln \sqrt[3]{\frac{x^2}{y^4(z+2)}}$$