3.5 EXPONENTIAL GROWTH AND DECAY

Turn to page 258.

Example 1: The population of a large city can be modeled by $y = 1.8e^{.026x}$, in millions, where x=0 corresponds to 1990. In what year is the population of this city expected to reach $\underbrace{2.5}_{y}$ million?

$$2.5 = 1.8e^{.026x}$$

$$1.389 = e^{.026x}$$

$$\ln 1.389 = \ln e^{.026x}$$

$$\ln 1.389 = .026x$$

$$\frac{\ln 1.389}{.026} = x$$

$$12.64 \approx x$$

$$\begin{array}{r} 1990 \\ +\ 12.64 \\ \hline 2002 \end{array}$$

2002, July

Answer: 2002

Example 2: The radioactive isotope ^{226}Ra has a half-life of 1620 years. If the original amount was 5 grams, how much would remain after 10,000 years?

use $y = ae^{bx}$

a = orig. amt.

b = rate of decay

x = time

y = amt. remaining after x years

$x = 1620$
$a = 5$
$y = 2.5$
$b = ?$

① Find b first (rate of decay)

$$2.5 = 5e^{b(1620)}$$

$$\frac{1}{2} = e^{1620b}$$

$$\ln\frac{1}{2} = \ln e^{1620b}$$

$$\ln\frac{1}{2} = 1620b$$

$$\frac{\ln\frac{1}{2}}{1620} = b$$

$$\boxed{-.000428 = b}$$

② Now find y when $x = 10{,}000$

$$y = 5e^{(-.000428 \times 10{,}000)}$$

$$\boxed{y = .0692 \text{ grams}}$$

LOGISTIC GROWTH MODLES

Example 3: State the game commission released 100 animals into a game preserve. The agency believes that the carrying capacity of the preserve is 1,200 animals and the growth of the herd can be modeled by

$$p(t) = \frac{1200}{1+8e^{-0.1588t}},$$

where t is the measured in months. How long will it take the herd to reach $\underbrace{\text{one half of the preserve's}}_{600 \text{ animals}}$ carrying capacity?

$$600 = \frac{1200}{1+8e^{-.1588t}}$$

$$\frac{600\left(1+8e^{-.1588t}\right)}{600} = \frac{1200}{600}$$

$$1+8e^{-.1588t} = 2$$

$$8e^{-.1588t} = 1$$

$$e^{-.1588t} = \frac{1}{8}$$

$$\ln e^{-.1588t} = \ln \frac{1}{8}$$

$$-.1588t = \ln \frac{1}{8}$$

$$t = \frac{\ln \frac{1}{8}}{-.1588}$$

$$\boxed{t = 13.095 \text{ months}}$$