9.1 Sequences and Series

An <u>infinite sequence</u> is a function whose domain is the set of positive integers.

x	1	2	3	4
$f(x)$				

position in the list

n	1	2	3	4	......
a_n					

term value

<u>Example:</u> Write the first four terms of the sequence

$$a_n = 2n + 5$$

n	1	2	3	4
a_n	7	9	11	13

or

$a_1 = 2(1)+5 = 7$

$a_2 = 2(2)+5 = 9$

$a_3 = 2(3)+5 = 11$

$a_4 = 2(4)+5 = 13$

Example: Write the apparent *n*th term of the following sequence: 0, 3, 8, 15 …

Make a table & create a rule

n	1	2	3	4
a_n	0	3	8	15

Ask yourself what operations can you do with the top row that generates the bottom row?

rule: $a_n = n^2 - 1$

A recursive sequence is a sequence where terms are found using the previous terms in the sequence.

Example: Write the first five terms of the sequence defined recursively.

$$a_1 = 32, \ \underbrace{a_{k+1}}_{\text{next term}} = \frac{1}{2} a_k$$

current term

k	1	2	3	4	5
a_k	32	16	8	4	2

$\frac{1}{2}(16)$ $\frac{1}{2}(8)$ $\frac{1}{2}(4)$

$$a_2 = \frac{1}{2} a_1$$
$$= \frac{1}{2}(32)$$

n factorial: $$n! = n(n-1)(n-2)\cdots 2\cdot 1$$

$(n-3)(n-4)$

$4! = 4\cdot 3\cdot 2\cdot 1 = 24$

$3! = 3\cdot 2\cdot 1 = 6$

$2! = 2\cdot 1 = 2$

$1! = 1 = 1$

$0! = 1$ (special case)

Example: Evaluate:

a) $$\frac{10!}{2!8!} = \frac{10\cdot 9\cdot \cancel{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}{(2\cdot 1)\cancel{(8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1)}} = \frac{\overset{5}{\cancel{10}}\cdot 9}{\cancel{2}\cdot 1} = 45$$

or $$= \frac{10\cdot 9\cdot \cancel{8!}}{2!\cancel{8!}} = \frac{\overset{5}{\cancel{10}}\cdot 9}{\cancel{2}\cdot 1} = 45$$

b) $$\frac{n!}{(n+1)!} = \frac{\cancel{n!}}{(n+1)\cancel{(n)!}} = \frac{1}{n+1}$$

c) $$\frac{(n-2)!}{(n-3)!} = \frac{(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = n-2$$

Summation notation: $\underbrace{\sum_{i=1}^{n}}_{\text{sigma notation}} a_i = a_1 + a_2 + a_3 + \cdots a_n$

Example: Find the following sum: $\sum_{i=1}^{4} 3^i + 1$

stop (4)

start ($i=1$)

In other words, find the sum of the first 4 terms!

$= a_1 + a_2 + a_3 + a_4$

$= (3^1 + 1) + (3^2 + 1) + (3^3 + 1) + (3^4 + 1)$

$= 4 + 10 + 28 + 82$

$= \boxed{124}$

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ptys. of sums